91. (Second problem in **Cluster 2**)
As explained in the previous solution, we take both angles θ_1 and θ_2 to be positive-valued.

(a) We first examine conservation of the y components of momentum.

$$\begin{aligned} 0 &= -m_1\, v_{1f}\, \sin\theta_1 + m_2\, v_{2f}\, \sin\theta_2 \\ 0 &= -m_1 v_{1f}\, \sin 30^\circ + 2m_1\, v_{2f}\, \sin\theta_2 \end{aligned}$$

Next, we examine conservation of the x components of momentum.

$$\begin{aligned} m_1\, v_{1i} &= m_1\, v_{1f}\, \cos\theta_1 + m_2\, v_{2f}\, \cos\theta_2 \\ m_1(10.0 \text{ m/s}) &= m_1 v_{1f}\, \cos 30^\circ + 2m_1\, v_{2f}\, \cos\theta_2 \end{aligned}$$

From the y equation, we obtain $v_{1f} = 4\, v_{2f}\, \sin\theta_2$; similarly, the x equation yields $20 - v_{1f}\sqrt{3} = 4v_{2f}\, \cos\theta_2$ with SI units understood (also, $\cos 30^\circ = \sqrt{3}/2$ has been used). Squaring these two relations and adding them leads to

$$v_{1f}^2\,(1+3) - 40v_{1f}\sqrt{3} + 400 = 16\, v_{2f}^2\,\left(\sin^2\theta_2 + \cos^2\theta_2\right)$$

and thus to $v_{2f}^2 = v_{1f}^2/4 - 5v_{1f}\sqrt{3}/2 + 25$. We plug this into the condition of total kinetic energy "conservation."

$$\begin{aligned} K_i &= K_f \\ \frac{1}{2}\, m_1\, v_{1i}^2 &= \frac{1}{2}\, m_1\, v_{1f}^2 + \frac{1}{2}\, m_2\, v_{2f}^2 \\ \frac{1}{2} m_1 \left(10\ \frac{\text{m}}{\text{s}}\right)^2 &= \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2}\,(2m_1)\left(\frac{v_{1f}^2}{4} - \frac{5\sqrt{3}}{2} v_{1f} + 25\right) \end{aligned}$$

This leads to an equation of second degree (in the variable v_{1f}):

$$\frac{3}{4}\, v_{1f}^2 - \frac{5\sqrt{3}}{2}\, v_{1f} - 25 = 0$$

which has a positive root $v_{1f} = \frac{5}{3}\sqrt{3}\left(1+\sqrt{5}\right) \approx 9.34$ m/s.

(b) We plug our result for v_{1f} into the relation $v_{2f} = \sqrt{v_{1f}^2/4 - 5v_{1f}\sqrt{3}/2 + 25}$ derived above and obtain $v_{2f} = \frac{5}{6}\sqrt{6}\left(\sqrt{5}-1\right) \approx 2.52$ m/s.

(c) Plugging these values of v_{1f} and v_{2f} into, say, the $v_{1f} = 4\, v_{2f}\, \sin\theta_2$ relation, we find $\theta_2 = 67.8^\circ$.